Delayed feedback control of noise-induced patterns in excitable media

A. G. Balanov,[1,2] V. Beato,[1] N. B. Janson,[3] H. Engel,[1] and E. Schöll[1]
[1]*Institut für Theoretische Physik, Technische Universität Berlin, Hardenbergstraße 36, D-10623 Berlin, Germany*
[2]*School of Physics and Astronomy, University of Nottingham, University Park, Nottingham NG7 2RD, United Kingdom*
[3]*Department of Mathematical Sciences, Loughborough University, Loughborough, Leicestershire LE11 3TU, United Kingdom*
(Received 28 November 2005; published 31 July 2006)

We show that characteristic features of noise-induced spatiotemporal patterns in excitable media can be effectively controlled by applying delayed feedback. Actually, by variation of the time delay and of the strength of the feedback one can deliberately change both spatial and temporal coherence, as well as adjust the characteristic time scales.

DOI: 10.1103/PhysRevE.74.016214 PACS number(s): 05.45.Gg, 05.40.Ca

Noise-induced pattern formation in excitable media has been attracting increasing attention, also for its potential importance for, and applicability in, neuroscience and cardiac dynamics [1].

It has been found that in such media external fluctuations are able to induce quite coherent spatiotemporal patterns [2,3], to maintain the existing patterns [4], and even to support wave propagation [5]. With respect to heart dynamics, for instance, it was suggested that chronic atrial fibrillation in heart tissue with pathologically decreased conductance might be due to the influence of fluctuations of the membrane voltage in heart cells [6].

In neurodynamics, Ca^{2+} waves triggered by noise were found in a network of glial cells (cells maintaining and supporting neurons) [7,8]. These waves can be important in generating correlated neural activity, while their patterns are proposed to be different in healthy and in pathological neural dynamics [8].

Under this perspective, in a broad area of science and technology it would be hugely appreciated to create instruments to manipulate the features of noise-induced patterns. Usually, control of dynamical systems means an adjustment of their essential dynamical properties like stability, coherence, timescales, etc., in a desirable manner imposing small perturbations. With this, the vast majority of methods for the control of complex motion developed so far are suitable for deterministic dynamics [9]. Examples include the control of pattern formation in catalytic CO oxidation on Pt(110) by global delayed feedback [10], feedback-mediated control of spiral waves in excitable media [11,12], or the recently proposed spatiotemporal delayed feedback method [13]. While the development of tools for control of noise-induced oscillation in simple finite-dimensional systems is in progress at the moment [14,15], not much is known yet about the *control of spatiotemporal patterns* generated by external fluctuations in spatially extended systems.

In this paper we study how delayed feedback affects the coherence of noise-induced spatiotemporal patterns in an excitable system. As a model we choose the Oregonator equations that describe the Belousov-Zhabotinsky (BZ) reaction: a famous paradigm of an excitable medium which is relatively easily accessible for an experiment. We simulate a realistic situation that reproduces the conditions of a real experiment that can be performed with this reaction.

We show that a feedback of the form $F(t)=g(v(t-\tau)-v(t))$, where $v(t)$ is the signal coming from the system and g is a scalar function, can be effectively used for the manipulation of essential properties of noise-induced spatiotemporal patterns. This feedback form was earlier proposed for the control of deterministic chaos [16] (including chaotic spatiotemporal patterns [17,18]).

We consider the photosensitive version of the Belousov-Zhabotinsky (BZ) reaction, which has become a prototype system for experimental studies of noise-induced phenomena in spatially extended excitable media (see, e.g., Refs. [3,5,19]). The photosensitive BZ reaction can be described by the modified Oregonator equations which have the following form [20]:

$$\partial_t u = \frac{1}{\epsilon}[u-u^2-w(u-q)] + D_u \partial_x^2 u,$$
$$\partial_t v = u - v, \tag{1}$$
$$\partial_t w = \frac{1}{\epsilon'}[fv+\phi-w(u+q)] + D_w \partial_x^2 w.$$

Here, variables u, v, w are concentrations of bromous acid, the oxidized form of the catalyst, and bromide, respectively. We fix $q=0.002$, $f=1.4$, $1/\epsilon=11.7$, $1/\epsilon'=1059$, $D_u=1$, $D_w=1.12$ [21]. Then, the kinetics ($D_u=D_w=0$) of the system is governed by the intensity of light incident on the medium, linked to the parameter ϕ in the model (1). With the increase of ϕ self-sustained oscillations in the system are suppressed, and for $\phi>4.24\times10^{-3}$ the system is excitable: The kinetics exhibits a unique stable steady state.

To model a photosensitive BZ medium subjected to fluctuating light intensity we introduce a stochastic component $\eta(x,t)$ into the parameter ϕ as

$$\phi(x,t) = \phi_0(1+\eta(x,t)). \tag{2}$$

We fix the parameter ϕ_0 at 0.005, for which the system is excitable. To be close to what is accessible in experiments with a photosensitive BZ medium, where the applied fluctuations of light are essentially the same within a certain small portion of the medium, we divide the spatial domain of length L (fixed and equal to 19.2) into N cells of equal size $\lambda=L/N$. To each cell number i, $i=1,2,\ldots,N$, correlated noise $\hat{\eta}_i(t)$ with zero mean is applied according to

$$\frac{d}{dt}\eta(x,t)|_{x\in[(i-1)\lambda,i\lambda]} \equiv \frac{d}{dt}\hat{\eta}_i(t) = \frac{1}{\tau_{ou}}[-\hat{\eta}_i + \xi_i(t)], \quad (3)$$

where $\xi_i(t)$ is Gaussian white noise. The correlation function of $\hat{\eta}_i(t)$ is $\langle \hat{\eta}_i(t)\hat{\eta}_j(s)\rangle = \delta_{ij}\sigma^2 \exp(-|t-s|/\tau_{ou})$. Here δ_{ij} denotes the Kronecker delta (0 if $i\neq j$ and 1 if $i=j$), τ_{ou} is the correlation time of the Ornstein-Uhlenbeck (OU) process described by Eq. (3) and $\sigma^2 = \langle \eta^2\rangle$ is its intensity which is chosen the same for all cells. There is no spatial correlation between fluctuations in different cells, so λ is a measure of the spatial correlation length.

Importantly, because ϕ is proportional to the applied illumination it cannot be negative. Therefore in the following we eliminate values of ϕ that are less than zero and larger than $2\phi_0$ in order to preserve symmetry in the noise distribution. Numerically, this is done as follows: we integrate Eq. (3) and at each time step we check if for all i the values of $\hat{\eta}_i(t)$ fall within the interval $[-1;1]$. If for some i this condition is not satisfied, we continue integration of Eq. (3) for the given i until a suitable value of $\hat{\eta}_i(t)$ occurs. When all $\hat{\eta}_i(t)$ are as required, we feed them into Eq. (1). Obviously, the stochastic process obtained in this way is no longer of OU type. However, we checked that correlation time and noise intensity of the stochastic process we used in our calculations deviate by less than 10% from the corresponding values of an OU process.

As noise is applied to the given point x of the medium, it takes on average t_a time units to excite the medium locally. t_a can thus be called activation time. After the point x of the medium achieves its excited state, it returns to its rest state during a "refractory period" t_r. During t_r no new nucleations are possible at the given position x. In the presence of spatial diffusion ($D_u\neq 0$, $D_w\neq 0$), at the excited point x of the medium a pair of pulse waves nucleate and then propagate in the opposite directions with the same constant velocity. In order to prevent nucleations on the border of the medium, for our study we adopt *periodic boundary conditions*, therefore the initiated waves eventually collide and annihilate.

In our work we fix the parameters of noise at the values that provide a maximally coherent spatiotemporal dynamics: $L=19.2$, $\sigma=0.5$, $\tau_{ou}=0.5$, and $\lambda=1.2$ [22]. At these parameters t_a is negligibly small.

A typical noise-induced pattern corresponding to the chosen set of noise parameters is shown in Fig. 1(a), where the values of $u(x,t)$ are shown by grey shading in logarithmic scale: larger values are marked by the darker shading.

In order to characterize the coherence of these spatiotemporal patterns, we introduce the space-averaged activator concentration of $u(x,t)$ [23,24],

$$\bar{u}(t) = \frac{1}{L}\int_0^L u(x',t)dx'. \quad (4)$$

Figure 1(b) shows a realization of $\bar{u}$ corresponding to the space-time plot displayed in Fig. 1(a). This realization exhibits pronounced spikes. For each spike number i, interspike intervals T_i can be introduced as the intervals between the successive crossings by the variable $\bar{u}(t)$ of a threshold 0.03 from above to below [see Fig. 1(b)]. Each spike number i persists within a certain time interval, Δ_i [25], during which the perturbation generated by the random input is propagated across the medium.

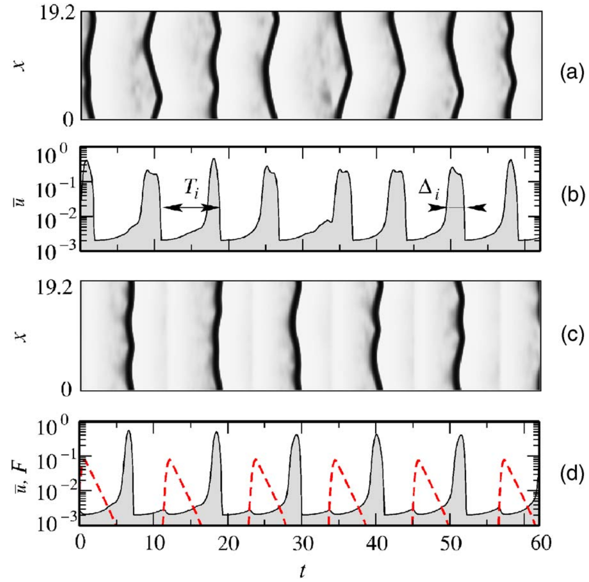

FIG. 1. (Color online) (a),(c) Space-time plots of $u(x,t)$ in logarithmic scale, (b),(d) spatially averaged inhibitor concentration $\bar{u}(t)$ (shaded), for the Oregonator model Eqs. (1). (a), (b) No delayed feedback. (c), (d) Delayed feedback force $F(t)$ as in Eq. (7) with $K=0.3$ and $\tau=5.25$ is applied to the whole medium [see white vertical stripes in (c)]. In (d) the control force $F(t)$ is also plotted (dashed line, red online).

The basic time scale of the noise-induced pattern can be characterized by the mean interspike interval $\langle T_i\rangle$ of $\bar{u}$, where $\langle\cdot\rangle$ denote the average over all spikes. The more temporally coherent the pattern is, the more periodically the quantity $\bar{u}$ spikes, i.e., the less the T_i's change from one spike to another. We characterize temporal coherence by the normalized variance of the interspike intervals

$$R_T = \sqrt{\langle (T_i - \langle T_i\rangle)^2\rangle}/\langle T_i\rangle. \quad (5)$$

The smaller the R_T, the more temporally coherent the oscillations are.

The average duration $\langle\Delta_i\rangle$ of the spikes in $\bar{u}$ characterizes the spatial homogeneity (coherence) of the patterns. If the pattern is close to being spatially homogeneous, at any time the values of concentration at different points in space are almost the same. Hence the spatially averaged concentration will behave almost as the local concentration at a single point: namely, it will exhibit large, pronounced spikes and quiescent periods between them. If the pattern is not spatially homogeneous, the spatially averaged concentration will become a smeared-out version of the concentration at a single point: the spikes will become lower and wider. The less homogeneous the pattern is, the wider and lower the spikes are in $\bar{u}$. Physically, homogeneity depends on the number of pulses nucleated in the medium at the same time. The more pulse pairs are nucleated almost simultaneously, the sooner they meet and annihilate, the narrower the spikes are in $\bar{u}$, and the smaller is $\langle\Delta_i\rangle$ [22]. Note that $\langle\Delta_i\rangle$ cannot be smaller

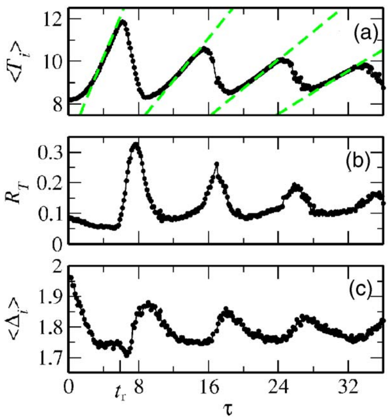

FIG. 2. (a) (Color online) Average interspike time $\langle T_i \rangle$, found numerically (black circles) and estimated analytically (grey-green online dashed line). (b) Normalized fluctuation R_T of the interspike intervals. (c) Mean duration of spikes $\langle \Delta_i \rangle$. Feedback strength is fixed at $K=0.2$.

than the duration of a pulse at a single point of the medium.

We now add time-delayed feedback to our system. Because in experiments with the photosensitive BZ medium the feedback is realized via the applied illumination, we introduce it into the parameter ϕ which then becomes

$$\phi(x,t) = \phi_0[1 + \eta(x,t)] + F(t), \tag{6}$$

where

$$F(t) = KsH(s), \quad s = v(x_0, t-\tau) - v(x_0, t). \tag{7}$$

Here $H(s)$ is the Heaviside function (0 for $s<0$ and 1 for $s \geq 0$), K denotes the feedback strength, τ is the time delay and $x_0=L/2=9.6$ is the detection point chosen arbitrarily. This form of the feedback force assures positive values for ϕ, which means that the control force can only suppress the activity in the system. Note that delayed feedback is nonlinear here. It is important to notice that the feedback signal is determined from the v field that is actually monitored in real experiments with the BZ medium.

The effect of delayed feedback on the noise-induced patterns is illustrated in Fig. 1(c). Here, $u(x,t)$ is shown in logarithmic scale by grey shading. White vertical stripes mark the areas to which the nonzero feedback force was applied with $\tau=5.25$ and $K=0.3$: these are the areas where the activity is suppressed by the feedback. Already from the space-time plots it is clear that the feedback can produce a remarkable effect on the system: the patterns are noticeably more regular than without control, the waves arriving at more equal time intervals. Moreover, it becomes more probable that more than one wave are initiated almost simultaneously, they collide sooner and thus result in patterns more homogeneous in space.

In order to gain a deeper insight into the effect of the feedback, we fix the feedback strength $K=0.2$ and study how the variation of the time delay τ influences the properties of noise-induced motion.

The three characteristics $\langle T_i \rangle$, R_T and $\langle \Delta_i \rangle$ of this motion depending on τ are given in Fig. 2. All the quantities display

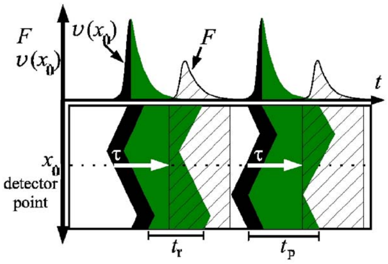

FIG. 3. (Color online) Bottom graph: a schematic space-time plot of noise-induced patterns: black areas indicate the excited state, grey (green online) areas the refractory state, and hatched areas indicate where the positive feedback is applied. Top graph: the respective profiles of $v(x_0)$ and of feedback force F.

a pronounced oscillatory behavior, with a characteristic time scale close to the mean interspike interval $\langle T_0 \rangle \simeq 8$ without feedback.

Mean interspike interval $\langle T_i \rangle$. This dependence contains almost linear segments [Fig. 2(a)], which can be associated with the entrainment of time scales by delayed feedback as recently discussed in Refs. [14,15] for systems without spatial degree of freedom. $\langle T_i \rangle > \langle T_0 \rangle$ holds for all plotted values of τ. This is because the feedback force is nonlinear and can only suppress the activity.

Temporal coherence R_T. Delayed feedback increases the temporal coherence of the noise induced patterns for $\tau \lessapprox 5$. R_T has a global minimum at $\tau \simeq 5$ [Fig. 2(b)]. Note that this minimum occurs at a value of τ close to the refractory period t_r (see Fig. 3).

Spatial coherence $\langle \Delta_i \rangle$. $\langle \Delta_i \rangle$ is less than $\langle \Delta_0 \rangle$, which is the value without feedback control, for all the plotted values of the feedback delay τ [Fig. 2(c)]. The feedback increases the number of simultaneous nucleations of wave pairs, and hence increases spatial coherence.

An important finding of this work is that the effect of the delayed feedback in the excitable medium described here is different from that of a system close to a Hopf bifurcation where maximal improvement of coherence was obtained for τ close to the mean period $\langle T_0 \rangle$ of oscillations without feedback. This difference between excitable and oscillatory systems was already noted for simple systems without spatial degrees of freedom in Refs. [14,15]. The behavior found in the present work is also markedly different from the linear delayed feedback control of noise-induced patterns in a globally coupled reaction-diffusion system used to model a semiconductor nanostructure [26].

We now focus on the mechanisms underlying the behavior of the system under delayed feedback control. Figure 3 sketches the patterns shown in Figs. 1(a) and 1(c). Here for simplicity we show single nucleations of counterpropagating pulses. From the steady state (white areas) an excitation starts (black stripes), followed by a refractory state (grey-green online areas) which lasts time t_r for each element of the medium. When this time has elapsed, each element recovers the steady state and can get excited again. The probability of a new nucleation is then proportional to the portion of the medium that has recovered the steady state. With the chosen noise parameters, a new nucleation occurs with probability close to unity immediately after the whole medium has passed the refractory period, but very often this happens even before the whole medium has fully recovered.

In the presence of feedback force, a positive light signal is applied globally, τ time units after the element at x_0 gets excited (hatched area in Fig. 3). This force inhibits activity in the medium resulting in an effective increase of the refractory period of the medium and thus of T_i. From this it follows directly that $\langle T_i \rangle$ depends linearly on τ when $\tau \leq t_r$. When the force vanishes, each element of the medium recovers the steady state simultaneously, so at this time the whole medium can get excited. This maximizes the probability of a nucleation and makes possible the emergence of highly spatially coherent patterns. Moreover, more than one nucleation becomes now more probable, which reduces the value of $\langle \Delta_i \rangle$.

Notably, if $\tau \lessapprox t_r$, the moment when the medium recovers globally to the steady state does not depend on the shape of the fronts of the spatiotemporal pattern. Hence the larger the τ, the stronger the positive feedback force is that suppresses the nucleation, the more effective the inhibition of activity is before the force vanishes. Therefore the maximal temporal and spatial coherence is achieved at τ close to t_r [compare the positions of the first minima of $R_T(\tau)$ and of $\langle \Delta_i \rangle$ in Figs. 2(b) and 2(c), respectively].

As τ gets closer to the mean period $\langle T_0 \rangle$ without control, the feedback is applied when the medium can already nucleate. So the force can suppress the activity even before the pulses meet and annihilate. This turns out to be the situation with the worst temporal and spatial coherence [see Fig. 2(a)].

When $\tau \geq \langle T_0 \rangle$ the situation described for $\tau \leq t_r$ is repeated, the coherence improves slightly and $\langle T_i \rangle$ increases. Moreover, it can be expected that for each successive linear segment in the dependence of the mean interspike interval $\langle T_i \rangle$ on τ the relationship

$$\langle T_i \rangle \approx (\tau + t_p)/n \tag{8}$$

holds, where n is integer and $t_p \approx 6.2$ is the average duration of the pulse $v(x_0, t)$ (excited state duration plus refractory period, see Fig. 3). As seen from Fig. 2(a), this expression can be used for quite an accurate estimate of $\langle T_i \rangle$ (grey-green dashed lines).

Finally, we checked the effect of feedback strength K, fixing τ at two characteristic values corresponding to the maximum and the minimum coherence. It was found that in both cases, as K grows, the action of the feedback becomes more prominent. Namely, if at the given value of τ the feedback increases (decreases) some characteristic quantity, the increase of K leads to the larger increase (decrease) of it.

In conclusion, by modeling a real experimental situation with a photosensitive BZ medium we have shown that a *nonlinear* time-delayed feedback is able to effectively manipulate the coherence of noise-induced spatiotemporal patterns. By choosing the appropriate time delay, one can deliberately increase or decrease both spatial and temporal coherence and adjust the time scales of the controlled dynamics. The same study was repeated using Neumann boundary conditions, under which the wave is absorbed by the boundary. The dependences for $\langle T_i \rangle$, R_T, and $\langle \Delta_i \rangle$ match those in Fig. 2 with high accuracy. An experimental verification of the predicted effect of delayed feedback control remains a challenge for future work.

This work was supported by DFG in the framework of Sfb 555. A.B. also acknowledges the support of EPSRC (UK).

[1] B. Lindner, J. García-Ojalvo, A. Neiman, and L. Schimansky-Geier, Phys. Rep. **392**, 321 (2004).
[2] J. García-Ojalvo, A. Hernández-Machado, and J. M. Sancho, Phys. Rev. Lett. **71**, 1542 (1993).
[3] V. Beato, I. Sendiña-Nadal, I. Gerdes, and H. Engel, Phys. Rev. E **71**, 035204(R) (2005).
[4] S. Alonso, I. Sendiña-Nadal, V. Pérez-Muñuzuri, J. M. Sancho, and F. Sagués, Phys. Rev. Lett. **87**, 078302 (2001).
[5] S. Kádár, J. Wang, and K. Showalter, Nature (London) **391**, 770 (1998).
[6] Y. Zhou and P. Jung, Europhys. Lett. **49**, 695 (2000).
[7] S. Coombes and Y. Timofeeva, Phys. Rev. E **68**, 021915 (2003).
[8] P. Jung, A. Cornell-Bell, K. S. Madden, and F. Moss, J. Neurophysiol. **79**, 1098 (1998).
[9] H. G. Schuster, *Handbook of Chaos Control: Foundations and Applications* (Wiley-VCH, New York, 1999).
[10] M. Kim, M. Bertram, M. Pollmann, A. von Oertzen, A. S. Mikhailov, H. H. Rotermund, and G. Ertl, Science **292**, 1357 (2001).
[11] V. S. Zykov, G. Bordiougov, H. Brandtstädter, I. Gerdes, and H. Engel, Phys. Rev. Lett. **92**, 018304 (2004).
[12] V. S. Zykov and H. Engel, Physica D **199**, 243 (2004).
[13] N. Baba, A. Amann, E. Schöll, and W. Just, Phys. Rev. Lett. **89**, 074101 (2002).
[14] N. B. Janson, A. G. Balanov, and E. Schöll, Phys. Rev. Lett. **93**, 010601 (2004).
[15] A. G. Balanov, N. B. Janson, and E. Schöll, Physica D **199**, 1 (2004).
[16] K. Pyragas, Phys. Lett. A **170**, 421 (1992).
[17] O. Beck, A. Amann, E. Schöll, J. E. S. Socolar, and W. Just, Phys. Rev. E **66**, 016213 (2002).
[18] J. Unkelbach, A. Amann, W. Just, and E. Schöll, Phys. Rev. E **68**, 026204 (2003).
[19] I. Sendina-Nadal *et al.*, Phys. Rev. Lett. **84**, 2734 (2000).
[20] H. J. Krug, L. Pohlmann, and L. Kuhnert, J. Phys. Chem. **94**, 4862 (1990).
[21] H. Brandtstädter, M. Braune, I. Schebesch, and H. Engel, Chem. Phys. Lett. **323**, 145 (2000).
[22] V. Beato and H. Engel, Fluct. Noise Lett. **6**, L85 (2006).
[23] V. Beato and H. Engel, Proc. SPIE **5114**, 353 (2003).
[24] C. Zhou and J. Kurths, Phys. Rev. E **65**, 040101(R) (2002).
[25] The peak duration Δ is given by the time the mean activator concentration satisfies the relation $\bar{u} \geq 0.03$.
[26] G. Stegemann, A. G. Balanov, and E. Schöll, Phys. Rev. E **73**, 016203 (2006).